increased over that with no mass injection. Hence the net radiant heating exhibits a minimum value.

In summary, it is concluded from this study that for the cold wall case, the radiative component of heat transfer is relatively insensitive to small blowing velocities but can be altered significantly at moderate to large blowing rates. The first-order radiation correction term of the convective heating was found to be greatly increased over the whole range of blowing considered.

For the hot wall case, the radiative correction term $G(\lambda)$ exhibits a maximum for a given Eckert number and blowing velocity. The total radiative component is altered over the entire range of blowing rates considered and a minimum value is found to exist [for a given $E$ and $f(0)$]. The first-order radiative correction term of the convective heating was found to be insensitive to small values of $f(0)$ while at moderate to large blowing rates, this correction term increased for small values of the Eckert number and decreased for large values of $E$ as the blowing increased. Additional results supporting this conclusion may be found in [3].

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# SHIELDING OF RADIATION BY SCREENS AND ITS SIMILARITY TO A GRAY GAS

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## NOMENCLATURE

$a$, a length given in Fig. 2;
$b$, a length given in Fig. 2;
$B$, black body intensity;
$d$, diameter given in Fig. 2;
$e$, black body emissive power;
$F_m$, a function defined by equation (4);
$G_m$, a function defined by equation (4);
$H_m$, a function defined by equation (6);
$I$, intensity of radiation;
$n$, number of screens;
$q$, heat flux;
$R$, radiosity.

Greek symbols

$\alpha$, absorptivity;
$\delta$, distance between two neighboring screens for equally spaced screens;
$\theta$, polar angle measured from normal, between $(I^+, \tau)$ or $(I^-, -\tau)$;
$\kappa$, volumetric absorption coefficient;
$\xi$, $\tau/\tau_\infty$;
$\tau$, optical thickness measured from lower wall, $= \alpha_n i$;
$\phi$, dimensionless emissive power $(e_i - R_w)/(R_\infty - R_w)$;
$\omega$, solid angle.

Subscripts and superscripts

$\infty$, upper wall;
$i$, screen number;
$k$, dummy index;
$m$, dummy index;
$n$, number of screens, also normal incidence;
$w$, lower wall;
$\theta$, in the $\theta$ direction;
$+$, positive $\tau$ direction;
$-$, negative $\tau$ direction.

## INTRODUCTION

THE REDUCTION of heat transfer between two surfaces by the interposition of multiple opaque shields is a well known technique. In the present analysis, shields constructed of black screens (i.e. shields with holes) are investigated. It is shown that a bundle of many screens act in a manner similar to a gray gas, giving rise to the familiar "jump" temperature near the wall [1]. However, contrary to the concept of a gray gas in radiative equilibrium, which is only an idealization of the real case (where conduction and convection take place), pure radiation through screens can be a real physical and practical situation.

## ANALYSIS

We consider a set of screens equally spaced between two walls as shown in Fig. 1. The screens are assumed to have a fine structure of uniform distribution of "holes" and are placed in random distribution relative to the location of the holes of one screen with respect to the other. A pencil of rays impinging on such a screen will be partially absorbed by the opaque region and will partially pass undisturbed through the "holes". Thus, statistically one may describe the interaction of incident radiation with a screen by considering a screen as a surface with absorptivity $\alpha_\theta$ and transmissivity $(1 - \alpha_\theta)$. In general, $\alpha_\theta$, the directional absorptivity, depends on the geometry of the screen. For simplicity the cases where

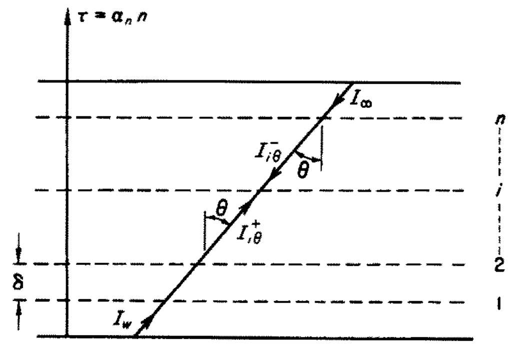


FIG. 1. Physical model.

$\alpha_\theta$ depends only on the polar angle measured from the normal and not the azimuthal angle, are considered. Emission in a certain direction also takes place from the opaque black region of the screen and is given by $\alpha_\theta B$, where $B$ is the black body intensity related to the screen temperature by Planck's Law.

A heat balance for a screen yields the following expression for the net radiated heat (see Fig. 1).

$$q_{i_{net}}{}^* = 2 \int_{2\pi} B_i \alpha_\theta \cos\theta \, d\omega - \int_{2\pi} \alpha_\theta I_{i\theta}^+ \cos\theta \, d\omega - \int_{2\pi} \alpha_\theta I_{i\theta}^- \cos\theta \, d\omega \qquad (1)$$

where

$$\left.\begin{aligned} I_{i\theta}^+ &= I_w(1-\alpha_\theta)^{i-1} + \sum_{k=1}^{i-1} \alpha_\theta B_k (1-\alpha_\theta)^{i-k-1} \\ I_{i\theta}^- &= I_\infty (1-\alpha_\theta)^{n-i} + \sum_{k=i+1}^{n} \alpha_\theta B_k (1-\alpha_\theta)^{k-i-1}. \end{aligned}\right\} \qquad (2)$$

Substituting equation (2) into (1) yields the following equation:

$$q_{i_{net}} = 4e_i F_0(\alpha_\theta) - 2R_w F_{i-1}(\alpha_\theta) - 2R_\infty F_{n-i}(\alpha_\theta) - 2\sum_{\substack{k=1\\k\neq i}}^{n} e_k G_{|i-k|-1}(\alpha_\theta) \qquad (3)$$

where

$$\left.\begin{aligned} F_m(\alpha_\theta) &= \int_0^{\pi/2} \alpha_\theta (1-\alpha_\theta)^m \cos\theta \sin\theta \, d\theta \\ G_m(\alpha_\theta) &= \int_0^{\pi/2} \alpha_\theta^2 (1-\alpha_\theta)^m \cos\theta \sin\theta \, d\theta. \end{aligned}\right\} \qquad (4)$$

For the case of radiative equilibrium it is convenient to non-dimensionalize the emissive power by $\phi_i = (e_i - R_w)/(R_\infty - R_w)$.

Equation (3) then takes the form

$$\frac{q_{i_{net}}}{R_\infty - R_w} = 4F_0\phi_i - 2F_{n-i} - 2\sum_{\substack{k=1\\k\neq i}}^{n} \phi_k G_{|i-k|-1}. \qquad (5)$$

For the case where heat is transferred only by radiation (radiative equilibrium) $q_{i_{net}} = 0$ and equation (5) results in $n$ linear equations for $\phi_1\, \phi_2 \ldots \phi_n$.

Once the temperature "profile", i.e. the temperature of every screen is given, the heat flux at the wall (and for radiative equilibrium at any position) is computed from:

$$\frac{q_w}{R_w - R_\infty} = 2H_n + 2\sum_{k=1}^{n} \phi_k F_{n-1}$$

where

$$H_m(\alpha_\theta) = \int_0^{\pi/2} (1-\alpha_\theta)^m \cos\theta \sin\theta \, d\theta. \qquad (6)$$

For black surfaces $R_w = e_w$ and $R_\infty = e_\infty$. For gray surfaces one needs to know $R_w - R_\infty$ in order to calculate $q_w$ and $e_i$. This can be done quite easily for the case of radiative equilibrium using equation (8-5) in [2]. However, before a solution of equation (5) can be obtained the directional absorptivity $\alpha_\theta$ must be known for the calculation of the coefficients, $F_m$, $G_m$ and $H_m$.

Figure 2 shows a typical screen section. From the geometry one can show that $\alpha_\theta$ for the particular direction shown on the figure is:

* $q_{i_{net}}$ is equivalent to the concept of the divergence of the heat flux used for continuous media [see equation (8)].

$$\left.\begin{aligned} \alpha_\theta &= \frac{[(b-d)\,a + b(a+b)]\cos\theta + ad}{(a+b)^2\cos\theta}, & \theta < \cos^{-1}\frac{d}{a+d} \\ \alpha_\theta &= 1, & \theta > \cos^{-1}\frac{d}{a+d}. \end{aligned}\right\} \quad (7)$$

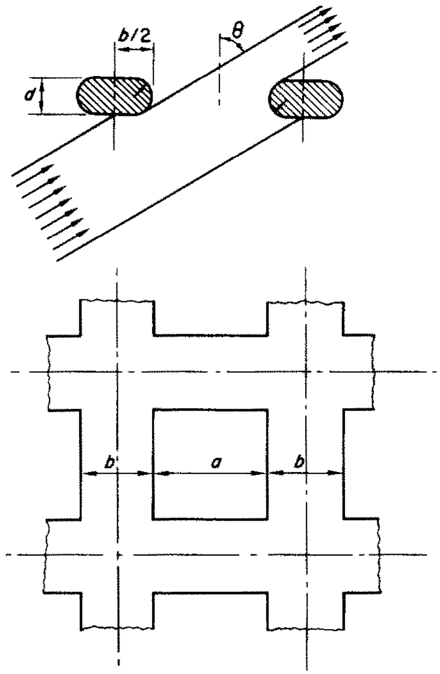


FIG. 2. The geometry of a typical screen.

Equation (7) includes the case where the screen wires are cylindrical, $b = d$, and the case of a flat screen $d \simeq 0$. In general, one can see that for this type of screen and for the two major directions aligned with the directions of the wires of the screen, the absorptivity consists of a term which is constant and a term proportional to $1/\cos\theta$. If the screen is flat ($d \simeq 0$) $\alpha_\theta$ is independent of the incident angle ("diffuse screen"). One may also visualize a screen consisting of beads equally distributed and connected by very thin wires. In this case it can be shown that $\alpha_\theta = \alpha_n/\cos\theta$ namely the absorptivity is proportional to $1/\cos\theta$. For this case $\alpha_\theta$ is also independent of the azimuthal angle.

## RESULTS

Solutions have been obtained for two representative cases: first, the case where $\alpha_\theta = \alpha_n$ = constant, and second, the case where $\alpha_\theta = \alpha_n$/c3s $\theta$($\alpha_\theta = 1$ for $\cos\theta < \alpha_n$). The first case corresponds to a "diffuse" character and represents the simplest case. The second resembles a gray gas for which the absorption of a small layer is given by $I_{\text{absorbed}} = -\kappa\Delta\tau/\cos\theta$.

For $\alpha_\theta = \alpha_n = \alpha$, one finds $F_m = \frac{1}{2}\alpha(1-\alpha)^m$, $G_m = \frac{1}{2}\alpha^2(1-\alpha)^m$ and $H_m = \frac{1}{2}(1-\alpha)^m$, and equation (3) can now be solved. For more than three screens a solution of the simultaneous equations becomes very cumbersome and a standard routine for the solution of the matrix equation $AX = B$, available in most computer centers, is used. When the number of screens is very large, $n \to \infty$, it can be shown that equation (3) reduces to the equation

$$q_{i_{\text{net}}} = \frac{dq}{d\tau} = 2e(\tau) - R_w \exp(-\tau) - R_\infty \exp[-(\tau_\infty - \tau)] - \int_0^{\tau_\infty} e(t)\exp[-(|\tau - t|)]\,dt \quad (8)$$

where $\tau$ is the optical distance defined as $\tau = \alpha_n i$. (This definition is similar to that used for a gray gas whose absorption coefficient is $\kappa = \alpha_n/\delta$.)

The solution to equation (8) proceeds in the same manner as the Kernel substitution solution for a gray gas ([2], p. 226). In fact, equation (8) can be considered as a possible case of the Kernel substitution approximation. The solution yields:

$$\phi = \frac{\tau_\infty\xi + 1}{2 + \tau_\infty} \qquad \frac{q_r}{R_w - R_\infty} = \frac{2}{2 + \tau_\infty}. \quad (9)$$

For the case $\alpha_\theta = \alpha_n/\cos\theta$ the coefficients $F_m$, $G_m$ and $H_m$ are calculated by expanding the term $(1 - \alpha_n/\cos\theta)^m$ in a binomial series, followed by a straight-forward integration. For large numbers of screens, summation can be replaced by integration, and equations (3–5) reduce to the well known gray gas equations [3].

Figure 3 shows the results for the temperature profile for both cases, namely $\alpha_\theta = \alpha_n$ and $\alpha_\theta = \alpha_n/\cos\theta$. Taking advantage of the anti-symmetry of the dimensionless emissive power profile about $\xi = \frac{1}{2}$, only half of the profile is shown for each case. It is clearly seen that the solution for increasing numbers of screens approaches the result for infinite number of screens (which is the gray gas solution when $\alpha_\theta = \alpha_n/\cos\theta$), giving rise to a temperature "jump" near the walls. Considering the solution for infinite screens as an approximation for a finite number of screens, one can see that this approximation is quite good in the thin case. Also this approximation is better for the case $\alpha_\theta = \alpha_n$ than for the case $\alpha_\theta = \alpha_n/\cos\theta$; probably, again, because the bundle of screens is more transparent to radiation from all directions in the first case than in the second case.

The dimensionless heat flux as a function of optical thickness and number of screens is shown in Fig. 4. Though the curves should be discontinuous, representing the value of heat flux at a discrete number of screens, it is convenient

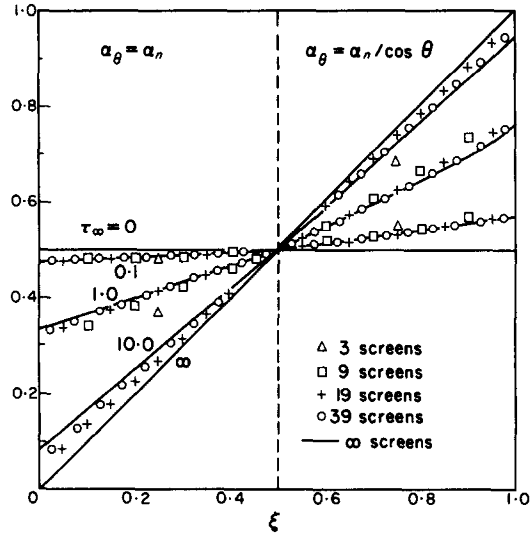


FIG. 3. Emissive power "profile".

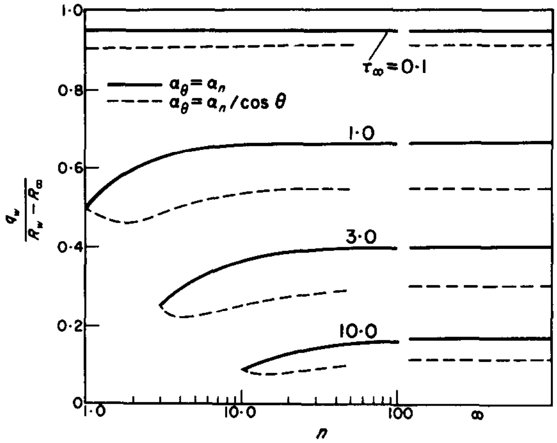


FIG. 4. Heat flux.

to draw them continuously. For each optical thickness, the minimum number of screens corresponds to the case where the screens are opaque, and for this case the solutions for $\alpha_\theta = \alpha_n$ and $\alpha_\theta = \alpha_n/\cos\theta$ both result in the same value, namely $q_w/(R_w - R_\infty) = 1/(n + 1)$. For non-opaque screens the heat flux is larger for the "diffuse" type because the hemispherical integrated transmissivity is larger. Generally, in both cases, better shielding is obtained when fewer, but more opaque, screens are used (keeping $\tau_\infty = \alpha_n n = \text{const.}$). However, for the case of $\alpha_\theta = \alpha_n/\cos\theta$ a dip in the heat flux curve appears, showing that one can get more effective shielding using, for example, four screens (of $\alpha_n = 0·75$) rather than three opaque screens; thus one may get more effective shielding using roughly the same amount of material. However, one should bear in mind that for shielding purposes one would use the best possible reflecting shields rather than the black ones considered here and only further analysis can determine whether the same effect occurs for reflective material.

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